

# How many commodity sectors are there, and how do they behave?

Geetesh Bhardwaj<sup>†‡</sup> and Adam Dunsby<sup>±</sup>

SummerHaven Investment Management

January 31, 2013

## Abstract

We find evidence for five commodity sectors that naturally conform to the standard functional categorizations typically used by the investment industry (industrial metals, energy, precious metals, grains & oilseeds, and livestock). Of the typical investment industry categorizations, only softs do not share a common factor. Using spot data to extend the history of commodity futures, we examine the performance of commodity sectors during periods of economic interest to investors and find 1) The industrial metals sector is very sensitive to economic conditions, while the grains & oilseed sector is insensitive. 2) Energy and precious metals are the sectors that earn the highest returns during periods of high and unexpectedly high inflation. 3) Precious metals do not do well when economic conditions are poor and do not outperform the typical commodity during those periods. 4) We show that commodities in general, and all commodity sectors, earn positive returns during US Dollar crashes.

**Key words:** Commodity futures, Factor analysis, Business cycle, Tail events

**JEL Classifications:** E31, F31, G11, G13

---

<sup>†</sup> Geetesh Bhardwaj, Director, SummerHaven Investment Management.

<sup>±</sup> Adam Dunsby, Partner, SummerHaven Investment Management.

<sup>‡</sup> Correspondence: SummerHaven Investment Management, LLC, Soundview Plaza, 1266 East Main Street, Stamford, CT 06902: 203-352-2704. Email: gbhardwaj@summerhavenim.com.

## Introduction

Investment professionals typically group commodities into sectors by function and physical form.<sup>1</sup> For instance, commodities that are burned as fuel (e.g. crude oil and natural gas) are categorized as energies, and row crops of the Midwest that are eaten (e.g. corn, wheat, and soybeans<sup>2</sup>) are categorized as grains. However, there are two problems with this approach. First it is unclear that some of the standard groupings make sense. For instance, coffee, sugar, and cocoa (and often cotton), are typically grouped into a softs sector. But why? Sugar and coffee are often used together, but sugar has many other uses as well. Or, is platinum best thought of as a precious metal or an industrial metal? Second, the standard categorizations may not be useful to investors. For instance, a tactical commodity investor is more likely to want to construct a portfolio of commodities that perform well during, say, high inflation than a portfolio of commodities that all grow on trees.

The return distributions of different sectors are indeed wide; table 1 presents the returns for Dow Jones-UBS commodity group indexes<sup>3</sup> and the range of sector returns (the difference in returns of the best and the worst performing commodity group). The range of sector returns is often quite wide. For instance, from January 2000 to December 2004 the Energy subindex outperforms the grain index by 218%. These divergences raise the prospect that different factors drive the different subindex returns and that the subindexes may offer opportunities to investors over and above those of broadly diversified commodity indexes.

The first part of this paper determines how many factors are needed to reasonably describe the twenty-five commodity return series, and how many, if any, of these factors can be interpreted as sectors, as commonly defined. A difficulty that must be addressed is the paucity of energy futures data prior to the 1980s. Of the four energy commodities examined, only heating oil extends back into the 1970s (1978). Given the prominence of

---

<sup>1</sup> In a brochure downloaded from the Dow Jones-UBS website on December 1, 2011, the "Sector Subindexes" are energy, petroleum, livestock, grains, industrial metals, precious metals, softs, and agriculture. "Petroleum" is a subset of "Energy," and "Agriculture" is a combination of "Grains" and "Softs" plus soybean oil (<http://www.djindexes.com/commodity/>). The S&P website (reference December 1, 2011) lists the sectors energy, industrial metals, precious metals, agriculture, livestock, softs, and grains (<http://www.standardandpoors.com/indices/sp-gsci/en/us/?indexId=spgscirg--usd---sp----->).

<sup>2</sup> Soybeans and, by extension soybean meal and soybean oil, may be more strictly categorized as oilseeds. For parsimony, we will typically refer to them as grains.

<sup>3</sup> DJUBS has many overlapping subindexes. These subindexes collect eighteen of the nineteen DJUBS commodities into non overlapping groups.

energy in contemporary commodity portfolios<sup>4</sup> and the volatility exhibited by commodities in the 1970s, we use spot commodity data—which is highly correlated with futures data—to extend our series back to 1970, where possible.<sup>5</sup>

We find that in an appropriately specified factor model, five of the factors are naturally interpreted as industrial metals, energy, precious metals, grains, and livestock; corresponding to standard industry categorizations. There is no factor that can be interpreted as a soft factor; and cotton, coffee, cocoa, and sugar do not cohere statistically.<sup>6</sup>

**Table 1 Dow Jones-UBS Commodity Group Indices**

5 Year Returns	Industrial Metals Subindex	Energy Subindex	Precious Metals Subindex	Livestock Subindex	Grains Subindex	Softs Subindex	Range
Jan-91 - Dec-94	7%	<b>-25%</b>	-12%	6%	-11%	<b>11%</b>	<b>36%</b>
Jan-95 - Dec-99	-24%	<b>37%</b>	-30%	-28%	<b>-32%</b>	-21%	<b>68%</b>
Jan-00 - Dec-04	28%	<b>194%</b>	25%	2%	<b>-23%</b>	-17%	<b>218%</b>
Jan-05 - Dec-09	68%	<b>-57%</b>	<b>107%</b>	-56%	7%	-16%	<b>164%</b>
Jan-10 - Dec-12	-12%	<b>-32%</b>	<b>58%</b>	3%	31%	6%	<b>90%</b>

Note: Table presents the returns for the six Dow Jones-UBS Commodity Group Indices. We also report the range of sector returns (the difference in returns of the best and the worst performing sector). Returns in blue highlight is best sector and ones in red highlight the worst sector. Source: Bloomberg.

Having identified the factors and their interpretation (in most cases) as the typically defined commodity sectors, the second part of the study examines the return distributions of the commodity sectors. Previous studies (e.g. GR 2006) have compared the return and correlation properties of a broad portfolio of commodity futures to other asset classes and how this portfolio relates to macroeconomic series such as inflation and the business cycle. Since commodities are broadly uncorrelated to one another, but statistically group into sectors, we seek to understand how the sectors compare to other asset classes and how they perform in different economic environments. For instance, a primary reason investors are interested in commodities is the positive correlation of their returns to inflation. It would

<sup>4</sup> The S&P GSCI, one of the most popular commodity indexes, has a weighting of over sixty percent (67.5%) in energy. The PowerShares DB Commodity Index Tracking Fund, one of the most popular commodity ETFs, has a weighting of 55% in energy.

<sup>5</sup> See Table 2 for details.

<sup>6</sup> As pointed out by an anonymous referee, an exploratory factor analysis, such as we have undertaken here, runs the risk of data mining (i.e. finding one of Fischer Black’s nuggets—Black, 1993). That we find a factor structure with a natural interpretation, we believe, leads to a real increased understanding of the structure commodity returns.

be interesting to know if this property is broadly shared across all commodities, or if it results from a subset of sectors.

We conclude that for an investor who wishes to achieve the “headline” benefits of commodity futures investments (equity-like returns, low correlation to equities, positive return correlation to inflation), a broadly diversified portfolio of commodities is adequate.<sup>7</sup> We also show that the identified commodity sectors have differing return profiles during certain economic tail events, such as recessions, providing guidance to investors who wish to implement tactical views or desire to structure commodity portfolios that are hedged or partially hedged against certain economic events.

We find that industrial metals are very sensitive to the state of the economy, earning poor returns during recessions and good returns during expansions. Grains, on the other hand, are relatively insensitive to the state of the economy. Energy, and to a lesser extent precious metals, do well both absolutely and relatively, during periods of both high inflation and high unexpected inflation. Precious metals do not provide a hedge against economic weakness. Commodities as a whole earn positive returns when the US Dollar declines, as does each of the individual sectors.

A trading strategy that is informed by these results is described in Bhardwaj and Dunsby (2012), which groups sectors into pro-cyclical (industrial metals and energy) and defensive (grains and softs). A simple trading strategy that switches between pro-cyclical and defensive sectors based on the available OECD composite leading indicators does better than the defensive, pro-cyclical or equally weighted sector portfolios.

## **Related Literature**

Factor analysis has been used extensively in the equity literature. Much of this relates to the testing of Ross’s arbitrage pricing theory (APT) such as in Roll and Ross (1980). A recent summary of the literature can be found in Connor and Koraczyk (2010).

Factor analysis has not been employed extensively in commodities. Kat and Oomen (2007) use the related tool of principal components to address the question of commodity sectors. Their analysis suggests the presence of energy, grain, coffee, pork, metals, sugar, cocoa and orange juice sectors.<sup>8</sup> They focus on the correlation of individual commodity returns

---

<sup>7</sup> Bodie and Rosansky (1980), Fama and French (1987), Gorton and Rouwenhorst (2006), and Erb and Harvey (2006) analyze returns for an equally weighted portfolio of commodity futures.

<sup>8</sup> Metals sector in Kat and Oomen (2007) analysis includes precious metals and copper, the paper covers the period of 1987-2005.

with the equity market under various macroeconomic scenarios rather than the investment performance of various commodity sectors.

Erb and Harvey (2006, EH) explore sector performance by analyzing the performance of five Goldman Sachs Commodity Index (GSCI) sectors (Energy, Livestock, Agriculture, Industrial Metals, and Precious Metals) for the period 1982 to 2004. They report considerable difference in sector returns over the period of their study, with energy sector returns of 7.06% to precious metal returns of -5.42%. EH find that while energy, livestock, and industrial metals are good inflation hedges, precious metals and agriculture sectors are not. Further, over their sample (1982 – 2004) EH find that only the energy and precious metal sector have significant negative beta to a trade weighted US Dollar index.

## Data

The commodities we study are those contained in either the Dow Jones-UBS Commodity Index or the S&P-GSCI (the former Goldman Sachs Commodity Index), omitting duplicates (e.g. we include WTI crude oil, but not Brent crude oil), and including tin, platinum, and soybean meal, based on the subjective assessment that they are important economically, have liquid futures markets, and are of interest to investors. The twenty-five commodities are listed in Table 2.

**Table 2 Commodities and start date for returns, January 1970 - December 2012**

Industrial Metals	Futures Returns Start Date	Spot Returns start date	Precious metals	Futures Returns Start Date	Spot Returns start date	Energy	Futures Returns Start Date	Spot Returns start date
Aluminum	July 1987	Jan 1970	Platinum	Jan 1970		Crude Oil	Apr 1983	Jan 1970
Copper	Jan 1970		Silver	Jan 1970		Gasoline	Jan 1985	Mar 1976
Nickel	May 1979		Gold	Jan 1975	Jan 1970	Heating Oil	Dec 1978	Jan 1970
Zinc	Feb 1977					Natural Gas	May 1990	Feb 1976
Tin	Aug 1989							
Lead	Mar 1977							
Grains	Futures Returns Start Date	Spot Returns start date	Softs	Futures Returns Start Date	Spot Returns start date	Livestock	Futures Returns Start Date	Spot Returns start date
Corn	Jan 1970		Cocoa	Jan 1970		Feeder Cattle	Dec 1971	Jan 1970
Soybean Oil	Jan 1970		Coffee	Sep 1972	Jan 1970	Lean Hogs	Jan 1970	
Soybean	Jan 1970		Sugar	Jan 1970		Live Cattle	Jan 1970	
Wheat	Jan 1970		Cotton	Jan 1970				
Soybean Meal	Jan 1970							

Source:

1. Futures returns for Copper and commodities in all the sectors except industrial metals are based on data from Commodity Research Bureau (CRB) and Bloomberg.
2. For industrial metals, other than copper, we use data from London Metals Exchange and Bloomberg.
3. Spot returns for Gold, Coffee, Crude Oil, Heating Oil, and Feeder Cattle are based on Commodity Research Bureau (CRB) spot prices. Spot Returns for Aluminum are based on Aluminum base scrap Producer Price Index series of the Bureau of Labor Statistics.

We construct monthly (excess) investment returns for futures by taking a long position at the end of each month in the nearest to expiry future that does not have its first notice date<sup>9</sup> or expiration date in the next month.

Table 2 lists the commodities in our sample and the start date of futures returns. While returns for commodities in livestock (except feeder cattle), softs and grains sectors go back to 1970, none of the energy contracts start until 1978. For industrial metals, only copper futures returns extend back to 1970. Gold future returns do not start until January 1975. It is not until May 1990 that all the commodities in our sample have futures returns.

Where possible, we extend the price series as far back as 1970 by adding to the beginning of the futures series spot data. We generate spot returns for gold, coffee, crude oil, heating oil, gasoline, natural gas, and feeder cattle using Commodity Research Bureau spot prices. For aluminum we use the Aluminum Base Scrap Producer Price Index series of the Bureau of Labor Statistics. For the purpose of factor analysis, the underlying assumption is that spot returns are expected to retain the correlation structure of futures returns. Table 3 displays the correlation between the monthly spot returns and futures returns for the eight commodities for which we use spot data, over the period for which we have both series. The correlations are generally high, ranging from a low of 64% for feeder cattle to a high of 98% for gold. Thus spot prices should be adequate in allowing us to extend back our series for these commodities.

**Table 3: Spot-futures correlation and volatility, January 1970 - December 2012**

	Aluminum	Gold	Coffee	Crude Oil	Gasoline	Heating Oil	Natural Gas	Feeder Cattle
<b>Correlation</b>	65%	98%	81%	97%	88%	85%	77%	64%

Note: Table displays the correlation between the monthly spot returns and futures returns, over the period for which we have both series.

In the next section we use factor analysis to explore whether any statistically identified factors can be interpreted as commodity sectors, as commonly defined.

### Factor analysis

Investors might conjecture that the the co-movement in commodity futures returns arises from a relatively few key economic factors like the business cycle, inflation, shifts in aggregate demand or supply, change in

<sup>9</sup> First Notice Day: The first day on which notices of intent to deliver actual commodities against futures market positions can be received.

inventories, and so forth. The premise of factor analysis is that a small number of unobservable common factors generate the co-movement within a multivariate system of observed time-series. Thus returns of individual commodity futures can be represented as a linear function of a small number of common factors and a commodity specific variance. Let  $X^1, \dots, X^n$  be the returns for  $n$  commodity futures and  $F^1, \dots, F^k$  be the  $K (< n)$  common factors. The factor model has the following representation:

$$x_t^i = a_i + \lambda_{i1}f_t^1 + \lambda_{i2}f_t^2 + \dots + \lambda_{ik}f_t^k + u_t^i, \quad i = 1, \dots, n; t = 1, \dots, T$$

where  $u_t^i$  captures the  $i^{th}$  commodity specific variance,  $\lambda_{ij}$  is called the factor loading of factor  $j$  for returns series  $i$ . Table 4 reports the p-values for the test of the null hypothesis for different numbers of factors over the period for which we have futures data for all commodities. The test suggests that for the 25 futures in our sample, a model with 8 factors is adequate. The null hypothesis for the test is that the  $k$ -factor model is adequate. The null hypothesis that 1, 2, ... 7 factor model is adequate is rejected by the data. The hypothesis that nine factors are needed to fit the data cannot be rejected at a 10% significance level while an eight factor model is borderline at the 1% significance level.<sup>10</sup>

**Table 4 Goodness-of-fit test for the number of factors, May 1990 to December 2012**

# of Factors	11	10	9	8	7	6	5	4	3	2	1
p-value	0.30	0.26	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: Table reports the p-values for the test of the null hypothesis that  $k$ -factor model is adequate. The test is carried out on a sample of 25 commodity returns, listed in Table 2. Data starts in May 1990, because it is not until May 1990 that all the commodities in our sample have futures returns. Source: see notes to Table 2.

Table 5 reports the results for the 8-factor model for the period May 1990 to December 2012 (the period for which all commodities have futures returns). The highlighted numbers show factor loadings (correlation between factor and the future return) that are greater than 0.4. For the eight factor model, the first factor represents the group of industrial metals; the second factor represents a group of energy commodities. The third factor is the precious metals group. Silver clearly behaves like a precious metal, while platinum also has a significant loading to the industrial metals factor. The fourth factor groups feeder cattle and live cattle. Factor 5 groups the grains (though soybean oil has a higher weighting on the sixth factor).

Table 5 also reports the results of a six factor model. The choice of a six factor model is an experiment to see how closely the resulting factors resemble the standard categorizations of industrial metals, energy, precious

<sup>10</sup> See appendix for detailed discussion of factor analysis and the goodness-of-fit test.

metals, grains, livestock, and softs. In the six factor model, the first factor group is industrial metals, second is energy, third is precious metals, fourth is livestock, and fifth is grains. The sixth factor is soybean oil. Interestingly softs (cocoa, coffee, sugar and cotton) do not form a group. Cotton has some loading on the grains factor; however cocoa, sugar and coffee do not load on any factor. These results suffer from a relatively short sample starting in May 1990.

**Table 5 Factor loadings, May 1990 to December 2012**

	Eighth Factor Model								Six Factor Model					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)
Aluminum	0.74	0.16	0.06	0.05	0.00	0.13	0.01	0.07	0.74	0.17	0.04	0.05	0.09	0.09
Copper	0.77	0.16	0.18	0.02	0.02	0.14	0.02	-0.02	0.76	0.16	0.18	0.04	0.12	0.08
Nickel	0.66	0.11	0.10	0.01	0.05	-0.03	0.11	-0.11	0.65	0.11	0.10	-0.02	0.09	-0.10
Zinc	0.72	-0.01	0.19	0.02	0.09	0.04	-0.07	0.03	0.72	-0.01	0.20	0.03	0.04	0.05
Tin	0.58	0.13	0.20	0.00	0.10	0.05	0.03	0.08	0.58	0.13	0.19	0.03	0.12	0.01
Lead	0.62	-0.01	0.12	-0.03	0.05	0.05	0.00	0.07	0.63	-0.02	0.12	-0.01	0.07	0.02
Platinum	0.40	0.10	0.57	0.09	0.08	0.08	0.04	-0.03	0.40	0.10	0.57	0.10	0.14	0.02
Silver	0.25	0.07	0.82	-0.03	0.07	0.02	0.01	-0.02	0.25	0.07	0.81	-0.02	0.10	-0.01
Gold	0.15	0.11	0.80	-0.05	0.06	0.02	0.02	-0.06	0.14	0.12	0.80	-0.06	0.09	-0.01
Cocoa	0.11	0.08	0.25	-0.03	0.13	0.10	0.01	0.07	0.11	0.08	0.25	-0.03	0.15	0.06
Coffee	0.17	-0.04	0.20	-0.03	0.10	0.07	0.07	0.12	0.17	-0.03	0.19	-0.01	0.14	0.00
Sugar	0.22	0.00	0.07	0.00	0.12	0.06	0.05	-0.04	0.22	0.00	0.07	-0.02	0.13	0.00
Cotton	0.26	0.06	0.09	-0.01	0.22	0.37	0.07	0.03	0.26	0.06	0.08	0.00	0.38	0.22
Crude Oil	0.16	0.93	0.11	0.06	0.00	0.06	0.00	0.30	0.18	0.92	0.08	0.07	0.04	0.06
Gasoline	0.18	0.86	0.10	0.08	0.03	0.02	0.01	0.06	0.18	0.87	0.09	0.10	0.04	0.02
Heating Oil	0.15	0.97	0.09	0.08	0.00	0.06	0.05	-0.09	0.14	0.95	0.08	0.09	0.07	0.02
Natural Gas	-0.02	0.45	0.03	-0.04	0.12	-0.01	0.02	-0.21	-0.03	0.45	0.05	-0.05	0.07	-0.03
Corn	0.10	0.04	0.15	-0.07	0.85	0.25	0.15	0.00	0.11	0.03	0.16	-0.14	0.69	0.03
Soybean Oil	0.16	0.02	0.13	0.02	0.36	0.84	0.13	0.00	0.16	0.02	0.10	0.06	0.76	0.55
Soybean	0.13	0.08	0.13	-0.03	0.51	0.59	0.59	0.02	0.11	0.08	0.09	0.00	0.98	0.04
Wheat	0.14	0.07	0.13	-0.01	0.61	0.17	0.10	0.00	0.14	0.06	0.14	-0.06	0.50	0.02
Soybean Meal	0.08	0.10	0.08	-0.07	0.53	0.25	0.79	0.00	0.06	0.11	0.03	-0.04	0.93	-0.34
Feeder Cattle	0.01	0.02	-0.07	0.85	-0.17	-0.01	0.09	0.02	0.00	0.01	-0.07	0.99	-0.07	-0.07
Lean Hogs	-0.02	0.07	0.02	0.37	0.01	0.02	-0.10	-0.05	-0.01	0.06	0.03	0.30	-0.06	0.07
Live Cattle	0.08	-0.03	-0.04	0.94	0.06	-0.03	0.05	0.09	0.08	-0.02	-0.04	0.80	0.02	-0.06

Note: Table reports the results for the 8-factor and 6-factor model for the period May 1990 to December 2012 (the period for which all 25 commodities listed in Table 2 have futures returns). The highlighted numbers show factor loadings (correlation between factor and the future return) that are greater than 0.4. Source: see notes to Table 2.

Tables 6 and 7 report the factor analysis results for the data set that includes spot prices starting January 1970. As discussed, by incorporating spot prices, we are able to extend our sample back to 1970 for six commodities, significantly extending the time that can be used to identify factors. In particular, the spot data allows us to analyze energy commodities prior to the 1980s. This data set contains 19 commodities. The results of longer dataset are consistent with the results in Tables 4 and 5; extending the sample back to 1970, factor analysis identifies five factors corresponding to the standard functional categorization: industrial metals, energy, precious metals, grains, and livestock.

**Table 6 Goodness-of-fit test for the number of factors, January 1970 to December 2012**

# of Factors	11	10	9	8	7	6	5	4	3	2	1
p-value	0.40	0.16	0.11	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00

Note: Table reports the p-values for the test of the null hypothesis that  $k$ -factor model is adequate. The test is carried out on a sample of 19 commodities, where future returns prepended by spot returns start in January 1970. See Table 6 below for the list of commodities. Source: see notes to Table 2.

**Table 7 Factor loadings, January 1970 to December 2012**

	Six Factor Model					
	(1)	(2)	(3)	(4)	(5)	(6)
Aluminum	0.61	0.08	0.11	0.08	0.07	-0.01
Copper	0.62	0.09	0.30	0.07	0.13	0.00
Platinum	0.27	0.09	0.72	0.09	0.11	-0.05
Silver	0.19	0.07	0.82	0.03	0.10	0.03
Gold	0.05	0.10	0.83	-0.01	0.13	0.00
Cocoa	0.20	0.09	0.20	-0.06	0.16	0.08
Coffee	0.15	-0.02	0.11	-0.04	0.19	-0.01
Sugar	0.19	-0.02	0.18	-0.04	0.16	0.04
Cotton	0.31	0.04	0.02	0.01	0.29	0.04
WTI Crude Oil	0.10	0.67	0.13	0.02	0.02	0.06
Heating Oil	0.07	0.99	0.07	0.02	0.06	-0.06
Corn	0.13	0.01	0.08	0.06	0.70	0.08
Soybean Oil	0.15	0.05	0.08	0.06	0.77	0.50
Soybean	0.07	0.03	0.12	0.10	0.97	-0.03
Wheat	0.17	0.06	0.10	0.08	0.49	0.06
Soybean Meal	0.01	0.02	0.12	0.11	0.91	-0.38
Feeder Cattle	0.07	-0.01	-0.02	0.85	-0.03	-0.03
Lean Hogs	-0.02	0.03	0.06	0.52	0.12	0.06
Live Cattle	0.05	0.00	-0.02	0.94	0.09	-0.06

Note: Table reports the results for the 6-factor model for commodity returns for the period January 1970 to December 2012. The analysis is carried out on a sample of 19 commodities, where future returns prepended by spot returns start in January 1970. The highlighted numbers show factor loadings (correlation between factor and the future return) that are greater than 0.4. Source: see notes to Table 2.

In summary, the standard categorizations of industrial metals, energy, precious metals, grains, and livestock are also factors. However, the commodities commonly categorized as softs do not form a factor.

### Sector returns

In the rest of the paper we analyze the return distributions of the commodity sectors. Among the issues we are interested is whether the identified factors/sectors behave differently conditional on various macroeconomic scenarios, potentially affording investors opportunities beyond what is provided by a diversified commodity portfolio.

Two decisions must be made. How should commodity futures within sector portfolios be weighted and which price series (i.e. futures and spot, or just futures) should be used. In regard to the first question, we form five commodity sector portfolios (industrial metals, energy, precious metals, grains, and livestock) consisting of equally weighted positions in all of the commodities within the sector available at that point in time. We believe these weightings are more intuitive, and they simplify analysis by avoiding small residual loadings. As can be seen in Table 8, the correlations between the portfolios with equal weights and the portfolios with factor loading weights are high. The correlations range from 0.81 for industrial metals to 0.94 for precious metals.

**Table 8 Correlation of factors and equally weighted sector portfolios, January 1970 to December 2012**

Industrial Metals	0.81
Energy	0.84
Precious Metals	0.94
Grains & Oilseed	0.94
Livestock	0.91

Note: Five commodity sector portfolios (industrial metals, energy, precious metals, grains, and livestock) consist of equally weighted, position in all of the commodities within the sector available at that point in time. We use the price series of futures pre-pended with spot returns, where available, thus lengthening the series, allowing all of the sector portfolios to reach back to the start date, January 1970. The factors are based on 6-factor model for commodity returns for the period January 1970 to December 2012, see Table 6 for details. Source: see notes to Table 2.

In the regard to the second question, we use the return series of futures pre-pended with spot returns, where available. By doing this we lengthen the series, allowing all the sector portfolios to reach back to the start date, January 1970, and capture the dynamic period of the early seventies. If we were not to do this, we would not have an energy sector until 1978. The consequence of this choice is that some of the “returns” are not futures returns, but percent changes in spot price. Given that change in futures prices and change in spot prices are highly correlated, we feel the ability to say more about the historical performance of commodity sectors outweighs the difficulties from using spot prices for some commodities in the early history<sup>11</sup>.

The sector return series we create are highly correlated with the commercially available commodity sectors returns provided by DJ-UBS and GSCI; notwithstanding that the commercial indices are futures only products, have different commodity weighting scheme (not equally

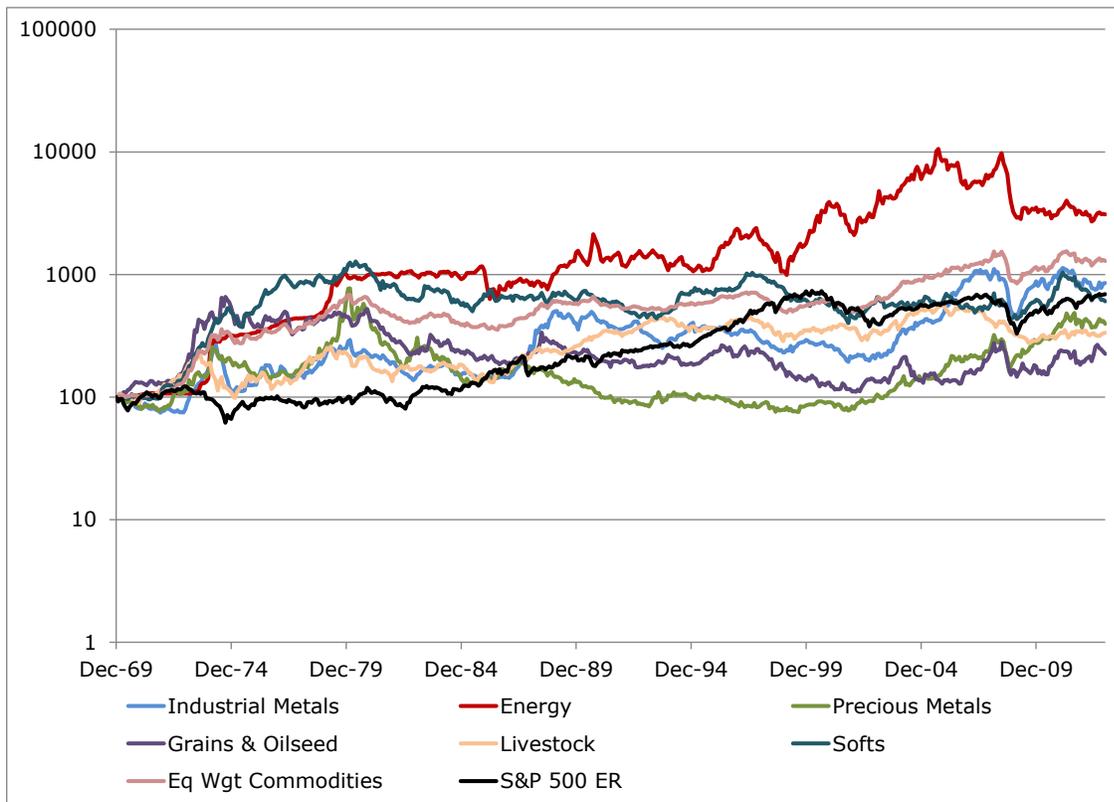
---

<sup>11</sup> The results presented in the following sections are robust to using sector returns based on futures only, however the analysis has to exclude the decade of 1970's.

weighted), and have different roll dates for futures. The correlation of our indices with DJUBS sector indices are in the range 0.94 to 0.99, while the same with GSCI sector indices range from 0.84 to 0.96.<sup>12</sup>

In what follows, we continue to present the softs sector, even though its components do not share a common factor, and are therefore perhaps best referred to as miscellaneous agriculture. We do this because the categorization is well established in the commodity investment industry.

**Figure 1 Cumulative excess returns of commodity sectors and S&P 500, January 1970 to December 2012**



Note: Commodity portfolios consist of equally weighted, position in all of the commodities within the sector available at that point in time. We use the price series of futures pre-pended with spot returns. Source: see Table 2 and Table 8 for details.

Figure 1 shows the cumulative excess performance of the six sectors and the equally weighted portfolio (Eq Wgt), along with the excess return of the S&P 500, from December 1969 through December 2012. All commodity sectors earn positive returns, the best being the energy sector and the worst being the grains. Two commodity sectors (industrial metals and energy)

<sup>12</sup> The monthly correlation of our energy, precious metals, industrial metals, livestock, grains, and soft sector returns with DJUBS sector returns are respectively 0.98, 0.94, 0.94, 0.96, 0.99, and 0.95 (while the same for SGCI sector indices are, 0.96, 0.95, 0.84, 0.95, 0.91, 0.92).

outperform the S&P 500, while three underperform (precious metals, grains & oilseed, and livestock).

**Table 9 Summary Statistics, January 1970 to December 2012**

	Geometric Returns	Arithmetic Returns	Volatility	Skew	Sharpe Ratio
S&P 500 ER	4.60%	5.74%	15.61%	-0.43	0.37
Bond ER	2.78%	2.93%	6.16%	0.30	0.48
US Dollar Index	-1.13%	-0.88%	7.08%	0.19	-0.13
Global Industrial Production	1.91%	1.92%	2.40%	-1.30	-
Unexpected Inflation	-0.77%	-0.77%	1.05%	-0.31	-
CPI	4.31%	4.23%	1.16%	0.08	-
One Month Treasury bill	5.11%	5.00%	0.88%	0.53	-
Industrial Metals	5.11%	7.18%	21.05%	0.38	0.34
Energy	8.30%	11.63%	27.84%	1.71	0.42
Precious Metals	3.26%	6.03%	24.00%	0.66	0.25
Grains & Oilseed	1.90%	4.67%	24.10%	1.11	0.19
Livestock	2.85%	4.28%	17.10%	-0.11	0.25
Softs	4.27%	6.09%	19.69%	0.53	0.31
Eq Wgt Commodity ER	6.13%	6.87%	13.53%	0.11	0.51

Note: Bond returns are composed of Ibbotson Associates SBBI Long US Term Government Total Returns from January 1970 to January 1976; and Barclays Capital US Aggregate Bond Index Total Returns since February 1976. Inflation is based on US CPI Urban Consumers SA (source: Bloomberg). Unexpected inflation is defined as the difference between inflation and one month Treasury bill returns. US Dollar index is a spliced return series of Trade Weighted Exchange Index: Major Currencies as calculated by Board of Governors of the Federal Reserve System (starting 1973) and DXY index (before 1973) which is composed of a basket of five currencies, Euro 57.6%, Yen 13.6%, Sterling 11.9%, Canadian Dollar 9.1%, Swedish Krona 4.2%, and Swiss Franc 3.6% (source: Bloomberg, and FRED). Global Industrial Production series is based on GDP weighted industrial production of the following countries United States, Germany, Japan, France, South Korea, Italy, UK, Canada, and Brazil (source: IMF and Bloomberg).

Table 9 presents summary statistics for the commodity sectors, the S&P 500, and other economic series of interest. The average annual return and volatility of the equally weighted index and stocks is similar, as has been previously established in Gorton and Rouwenhorst (2006, GR). The Sharpe ratio results are interesting. The equally weighted commodity portfolio has a higher Sharpe ratio than any of the commodity sector portfolios. From a Sharpe ratio perspective, any return the equally weighted portfolio gives up is made back from the decrease in volatility from increased diversification.

The skew results reveal that, consistent with previous research (GR), the skew on stocks is negative while the skew on the equally weighted index is positive (though over this time period, it is only modestly positive). With the exception of livestock, all the commodity sector skews are positive. The largest skew is energy (1.71).

Table 10 displays the average annualized basis (over the period for which futures data is available).<sup>13</sup> As shown in GR, the average basis is negative. Of the six portfolios, energy and livestock have positive average annualized bases. The most negative basis is grains, consistent with these commodities having an influx of supply at the harvest then having to be stored throughout the year. Energy, livestock, and industrial metals have the highest average bases, consistent with these commodities either being difficult to store or storage not necessary (i.e. they can be left in the ground).

**Table 10 Annualized Basis - January 1970 - December 2012**

	Average Basis
Industrial Metals	-2.25%
Energy	0.58%
Precious Metals	-5.79%
Grains & Oilseed	-6.20%
Livestock	0.33%
Softs	-4.62%
Eq Wgt Commodity	-5.26%

Note: We calculate basis calculated for each commodity as  $(F1/F2 - 1) * 365 / (D2 - D1)$ , where F1 is the nearest futures contract and F2 is the next nearest futures contract; D1 and D2 are the number of days until the last trading date of the respective contracts. Source: see notes to Table 2.

Table 11 displays the annual correlations of the commodity sectors with stocks, bonds, and selected economic and financial series: a US Dollar index, global industrial production<sup>14</sup>, unexpected inflation<sup>15</sup>, and inflation. The correlation of stocks and the equally weighted commodity index is 0.01, reproducing the well-known result that stocks and commodities have low correlation. The correlation between the US Dollar index and the equally weighted index is -0.42, demonstrating that a diversified portfolio of commodity futures earns a higher than average return during periods of a declining US Dollar. The correlation between the equally weighted index and global industrial production is 0.52. The same correlation for stocks is 0.30. Both commodity and stock returns exceed their average when industrial expansion is strong.

<sup>13</sup> Gorton Hayashi and Rouwenhorst (2007) link the basis to the level of inventories, following their analysis we calculate basis calculated for each commodity as  $(F1/F2 - 1) * 365 / (D2 - D1)$ , where F1 is the nearest futures contract and F2 is the next nearest futures contract; D1 and D2 are the number of days until the last trading date of the respective contracts.

<sup>14</sup> Global Industrial Production series is based on GDP weighted industrial production of the following countries United States, Germany, Japan, France, South Korea, Italy, UK, Canada, and Brazil.

<sup>15</sup> Following, Fama and Schwert (1977) and Gorton and Rouwenhorst (2006), we use short-term T-bill rate as a proxy for the market's expectation of inflation. The unexpected inflation can thus be defined as the actual inflation minus the short-term T-bill rate.

The correlation of the equally weighted commodity index to unexpected inflation and inflation is 0.48 and 0.29, respectively. For stocks, these correlations are -0.17 and -0.22. This is the known result that in higher than average inflation environments a diversified portfolio of commodity futures earns a return that exceeds its average, while stocks underperform its average.

Of interest in this study are how the sectors correlate to important financial and macroeconomic series, and how these correlations compare with those of the equally weighted commodity futures portfolio. The basic result is that the individual commodity sectors correlate to the financial and macroeconomic series with a profile very similar to that of the equally weighted index. As reported previously, the correlation between the equally weighted commodity futures portfolio and stocks is 0.01. The correlations of the five commodity sectors (and softs) with stocks do not vary much, ranging from -0.14 to 0.18. This lack of deviation also holds for the other financial and economic series.

In the investment community, precious metals are often viewed as hedges against various economic ills such as high inflation, economic recession, and a falling US Dollar. The correlation between precious metals and industrial production is 0.21, a number inconsistent with a notion that precious metals offer a hedge against economic downturn. The correlations between precious metals and the US Dollar index, unexpected inflation, and inflation are -0.38, 0.53, and 0.25, respectively. The corresponding numbers for the equal-weight index are -0.42, 0.48, and 0.29. Thus precious metals don't offer investors any better correlation properties with respect to the US Dollar and inflation than the equally weighted index.

**Table 11 Annual Correlations, January 1970 – December 2012**

	S&P 500 ER	Bond ER	Dollar Index	GDP Weighted GLOBAL IP	Unexpected Inflation	CPI	Industrial Metals	Energy	Precious	Grains & Oilseed	Livestock	Softs	Equally Weighted Commodity ER
S&P 500 ER	1	0.32	-0.16	0.30	-0.17	-0.22	0.08	-0.14	0.07	-0.08	0.18	-0.06	0.01
Bond ER	0.32	1	-0.10	-0.19	-0.33	-0.56	-0.32	-0.34	-0.25	-0.23	-0.01	-0.21	-0.36
Dollar Index	-0.16	-0.10	1	-0.23	-0.28	0.09	-0.37	0.00	-0.38	-0.32	-0.26	-0.28	-0.42
GDP Weighted GLOBAL IP	0.30	-0.19	-0.23	1	0.00	0.01	0.53	0.18	0.21	0.35	0.39	0.36	0.52
Unexpected Inflation	-0.17	-0.33	-0.28	0.00	1	0.39	0.23	0.35	0.53	0.31	0.09	0.44	0.48
CPI	-0.22	-0.56	0.09	0.01	0.39	1	0.10	0.45	0.25	0.14	-0.05	0.26	0.29
Industrial Metals	0.08	-0.32	-0.37	0.53	0.23	0.10	1	0.25	0.43	0.52	0.23	0.42	0.75
Energy	-0.14	-0.34	0.00	0.18	0.35	0.45	0.25	1	0.36	0.25	0.02	0.36	0.56
Precious	0.07	-0.25	-0.38	0.21	0.53	0.25	0.43	0.36	1	0.42	0.12	0.49	0.68
Grains & Oilseed	-0.08	-0.23	-0.32	0.35	0.31	0.14	0.52	0.25	0.42	1	0.20	0.61	0.81
Livestock	0.18	-0.01	-0.26	0.39	0.09	-0.05	0.23	0.02	0.12	0.20	1	0.00	0.33
Softs	-0.06	-0.21	-0.28	0.36	0.44	0.26	0.42	0.36	0.49	0.61	0.00	1	0.74
Equally Weighted Commodity ER	0.01	-0.36	-0.42	0.52	0.48	0.29	0.75	0.56	0.68	0.81	0.33	0.74	1

Note: Table displays the correlations of rolling annual returns. Source: see Table 2 and Table 8 for details.

## Commodity sector returns and tail events

In this section, we explore how commodity sectors perform during recessions, stock crashes, US Dollar crashes, high inflation, and high unexpected inflation. For recessions, we use the NBER categorization. There are seven recessions during the period January 1970 through December 2012. Eighty-three out of the 516 categorized months are recession months.

In defining tail events for the other economic shocks, we must decide how big to make the tails. The trade-off is between how extreme the events are and how many months are included. For instance, a one percent tail would certainly consist of extreme events but, in our sample, would only encompass five months. A bigger tail would include more months, but each month added would be less extreme. We decide to use five percent tails, which leads to a tail size of twenty six months (for the non-recession variables).

We analyze the performance of commodity sectors during tail events in two ways that we believe are important to investors. First we calculate the average monthly return of the sector during the tail event months. Both tactical investors and buy-and-hold investors are concerned with an asset's expected return during extreme economic events. Second, we calculate the same average monthly return for each sector and subtract from it the average return of an equally weighted portfolio consisting of the commodities *not* in the sector. This addresses the issue of whether certain commodity sectors behave differently from the typical commodity during tail events.

### Recessions

Panel A.1 of Figure 2 displays the excess monthly returns of the equally weighted portfolio and commodity sector portfolios during recessions. The equally weighted portfolio has a negative average return, as do four of the six sectors. The industrial metals sector average return is negative (-1.81%) and statistically significant (at a 10%, two sided, level). Both the grain and softs returns are very nearly zero.

Panel A.2 (Figure 2) displays the difference between each sector and the commodities not in that sector. Industrial metals underperform the non-industrial metals by -1.55%, and this difference is statistically significant.

### Stock crashes

Panel B.1 (Figure 2) displays monthly returns during the 5% of months with the lowest stock returns. The equally weighted portfolio and five of the sectors are negative. The average monthly return of grains is 1.15%. The

average monthly return of industrial metals is -4.47%, which is statistically significant.

As shown in Panel B.2 (Figure 2), grains outperform the non-grains 3.13% per month during stock crashes (though not statistically significant). Industrial metals underperform non-industrial metals by -4.00%, which is statistically significant.

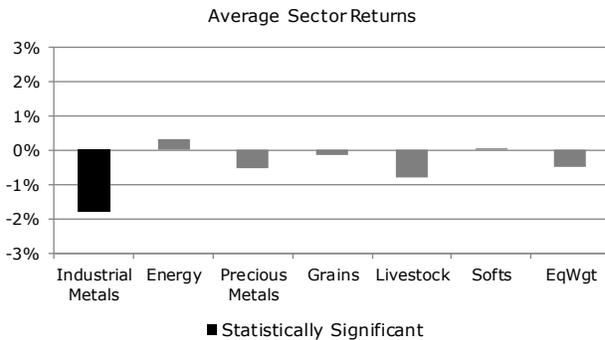
### US Dollar crashes

Panel C.1 (Figure 2) displays average monthly returns during the 5% of months in which the US Dollar declines the most. The equally weighted portfolio earns an average of 2.99% per month, and all sector returns except Livestock (-0.01%) are positive. All of these average returns are statistically significant, with the exception of energy and livestock. Panel C.2 (Figure 2) shows that the differences are dispersed around zero with only livestock's difference statistically significant (-3.43%).

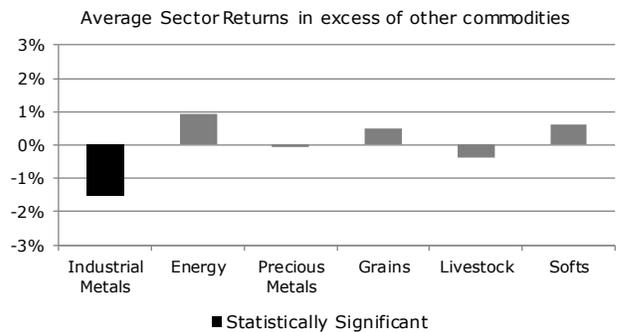
**Figure 2 Commodity returns during on Tail events, January 1970 to December 2012**

#### A.1

**Event: Recession**

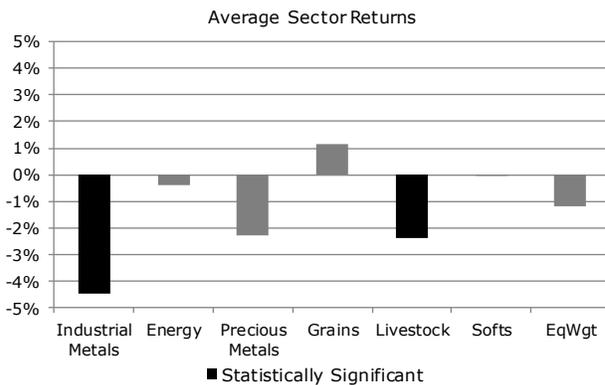


#### A.2

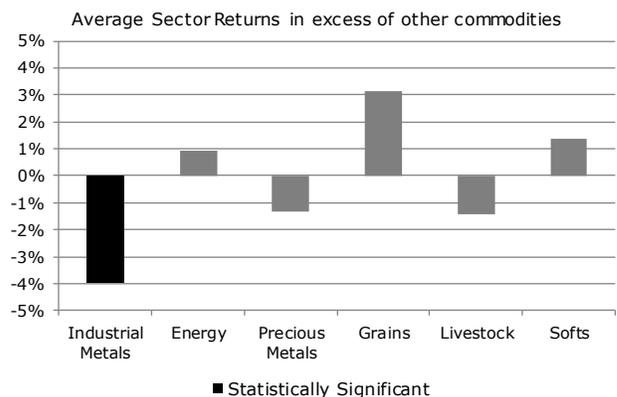


#### B.1

**Event: Stock Crash**

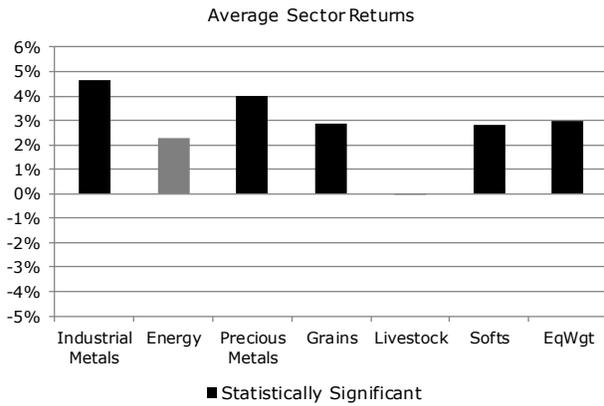


#### B.2

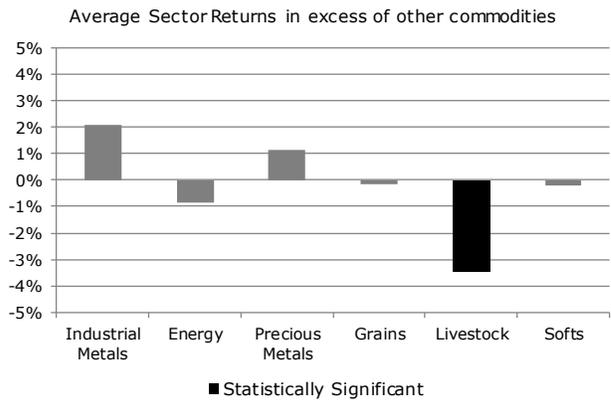


### C.1

**Event: Dollar Crash**

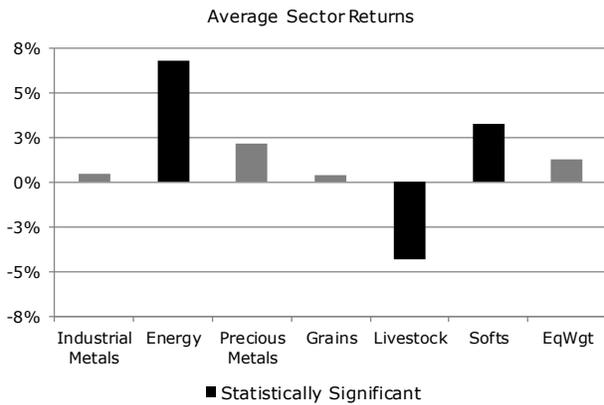


### C.2

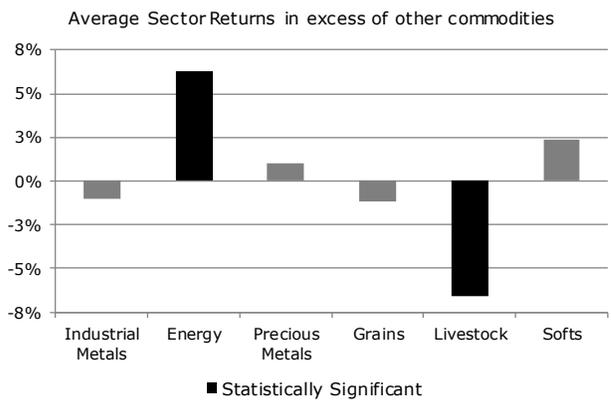


### D.1

**Event: High Inflation**

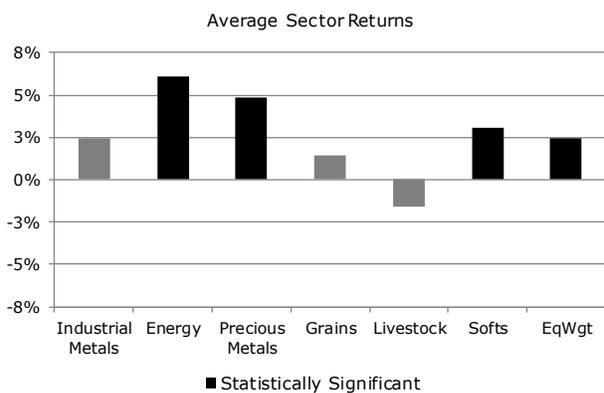


### D.2

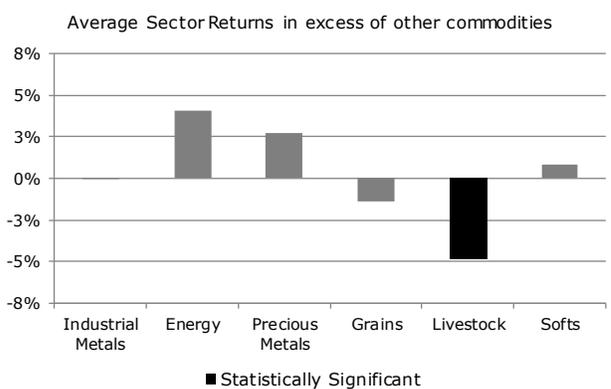


### E.1

**Event: High Unexpected Inflation**



### E.2



Note: For every event (recession, stock/bond crash, high inflation) table reports the commodity sector returns. We also report sector returns in excess of average return of an equal-weight portfolio consisting of the commodities not in the sector. Highlighted numbers are statistically significant at a 10%, two sided, level. Source: see Table 2 and Table 8 for details

## High inflation

Panel D.1 (Figure 2) displays monthly average returns during the 5% of months in which inflation is the highest. The average monthly return of the equal-weight index is 1.31%. The energy return is a particularly large 6.80%. Grain and livestock returns are negative, with livestock returns significantly different from zero.

Panel D.2 (Figure 2) displays the return differences between the sectors and the equally weighted portfolio. The energy sector outperforms non-energies by 6.30% and the livestock sector underperforms the other sectors by -6.59%, which is statistically significant.

## High unexpected inflation

Unexpected inflation is defined as the difference between the monthly percent change in the CPI index and the monthly one-month Treasury bill return.

Panel E.1 (Figure 2) displays average monthly returns during the 5% of months in which unexpected inflation is the highest. The equally weighted portfolio and five of the sectors earn positive returns. The equally weighted, energy, precious metals, and softs returns are statistically significant.

As seen in Panel E.2 (Figure 2), energy outperforms non-energy by 4.07% and precious metals outperform non-precious metals by 2.73%, though neither is significant at the 10% level. Livestock underperforms the portfolio of non-livestock by -4.85%, which is statistically significant.

## Summary of tail event results

Industrial metals are highly sensitive to events associated with economic weakness (recessions and stock crashes), and much more so than the typical commodity. Grains, on the other hand, are relatively insensitive to economic weakness. Grains earn about zero during recessions and positive returns during stock market crashes. From an asset pricing theory perspective, this is consistent with unconditional annual geometric returns of industrial metals of 5.11% and grains of 1.90%. Asset pricing theory requires that assets that earn low returns during bad times should earn higher unconditional returns. The energy sector earns very high returns during periods of both high inflation and high unexpected inflation. To a lesser extent, precious metals also do well during high inflation, high unexpected inflation periods.

It is worthwhile to discuss the performance of precious metals since they are an asset class that investors often expect to provide protection

during bad times. During economic weakness (recessions and stock crashes) precious metals do not earn positive returns and they do not outperform the typical commodity. During stock crashes, precious metals underperform non-precious metals by over 1.3% per month. Precious metals do earn significantly positive returns during US Dollar crashes, as do all commodity sectors. As previously discussed, precious metals do earn positive returns during periods of both high and unexpectedly high inflation. Summarizing, historically, precious metals have not earned positive returns during periods of economic weakness and have not outperformed the typical commodity during these periods. Precious metals do earn positive returns during periods of both high and unexpectedly high inflation, and they do outperform the typical commodity, though the results are not statistically significant.

### Commodity sector returns and the stages of the business cycle

This section explores in more depth the returns of commodity sectors during various stages of the business cycle. Table 12 displays the monthly returns of commodities and stocks during various stages of the business cycle<sup>16</sup>.

**Table 12 Business cycle analyses, January 1970 – December 2012**

Monthly returns and stages of the business cycle							
	Whole	Recession			Expansion		
	Sample	All	Early	Late	All	Early	Late
S&P 500 ER	0.48%	-0.60%	-2.40%	1.24%	0.68%	0.74%	0.49%
Bond ER	0.24%	0.49%	-0.13%	1.13%	0.20%	0.38%	-0.06%
US Dollar Index	-0.07%	0.13%	0.08%	0.19%	-0.11%	-0.23%	0.00%
CPI	0.35%	0.45%	0.57%	0.33%	0.33%	0.30%	0.40%
Global Industrial Production	0.16%	-0.56%	-0.27%	-0.85%	0.30%	0.29%	0.31%
Industrial Metals	0.60%	-1.81%	-1.21%	-2.42%	1.06%	0.45%	1.69%
Energy	0.97%	0.34%	2.99%	-2.38%	1.09%	0.91%	1.54%
Precious Metals	0.50%	-0.53%	-1.58%	0.55%	0.70%	0.06%	1.15%
Grains & Oilseed	0.39%	-0.14%	0.47%	-0.76%	0.49%	0.02%	0.86%
Livestock	0.36%	-0.79%	-0.70%	-0.88%	0.58%	0.48%	0.73%
Softs	0.51%	0.01%	0.44%	-0.43%	0.60%	0.77%	0.41%
Eq Wgt	0.57%	-0.50%	0.06%	-1.06%	0.78%	0.45%	1.12%

Note: Table displays the monthly returns of commodities and stocks during various stages of the business cycle. We divide the individual recession/expansion periods in equal halves to define early and late recession/expansion. The period of expansion subsequent to June 2009 is not categorized as either early or late expansion.

Source: See Table 2 and Table 8 for details. Business cycle dates are based on National Bureau of Economic Research's Business Cycle Dating Committee.

<sup>16</sup> Note that these returns do not reflect the returns of a trading strategy as the NBER dates business cycles after the fact.

Previous research has linked business cycle conditions to stock and bond returns (see Fama and French (1989)). GR have shown that a portfolio of commodity futures returns, like stocks, earns higher returns during expansions than recessions, but, unlike stocks, commodity futures earn a higher return in the early stage of a recession than in the late stages. These results are reproduced in Table 11.<sup>17</sup>

Each of the commodity sectors, with one exception, earns lower returns in the late stage of the business cycle than in the early stage, as is true for the equally weighted portfolio. This may reflect the build of inventories late in the business cycle.<sup>18</sup> The exception is precious metals, which earn a 0.55% per month late in the business cycle and -1.58% early in the business cycle. It is possible this reflects investors moving into precious metals as a recession lengthens and concerns about a deep collapse increase.

## **Conclusion**

A factor analysis identifies five factors corresponding to the standard functional categorization: industrial metals, energy, precious metals, grains, and livestock. We do not find a softs factor. Coffee, sugar, cocoa, and cotton do not cohere to a common factor.

Broadly speaking, for investors interested in the “headline” benefits of commodity investing (equity-like returns, low correlation with equities, positive correlation with inflation) a diversified portfolio of commodity futures does a good job of delivering those properties. Some sectors, such as energy, produce higher returns but they do so with higher volatility. The equally weighted index produces a higher Sharpe ratio than any individual commodity sector.

The commodity sectors do exhibit different behavior during macroeconomic and financial tail events. Industrial metals are particularly sensitive to economic weakness, producing low returns during recessions and stock crashes. During economic expansions, industrial metals produce high returns. Grains, on the other hand, are relatively insensitive to the economy, actually producing positive returns during stock market crashes. Energy produces high returns during periods of both high inflation and high unexpected inflation.

---

<sup>17</sup> We divide the individual recession/expansion periods in equal halves to define early and late recession/expansion.

<sup>18</sup> See Gorton, Hayashi and Rouwenhorst (2007) for details.

Precious metals do not produce positive returns during periods of economic weakness, and do not outperform the typical commodity. They do produce positive returns during periods of both high inflation and high unexpected inflation, and they do as well or better than the typical commodity.

We also find that a diversified portfolio of commodity futures produces significantly positive returns during periods of extreme US Dollar weakness, and all of the commodity sectors produce positive returns.

## References

- Bhardwaj, Geetesh and Adam Dunsby (2012), Commodities Sectors and The Business Cycle, *Journal of Indexes* 10-13.
- Black, Fischer (1993), Estimating Expected Return, *Financial Analysts Journal*, 49-5: 36-38.
- Bodie, Zvi and Victor Rosansky (1980), Diversification Returns and Asset Contributions, *Financial Analysts Journal* 26-32.
- Connor, Gregory and Robert Korajczyk (2010), Factor models of asset returns. *Encyclopedia of Quantitative Finance*, Rama Cont, ed., Wiley.
- Erb, Claude and Campbell Harvey (2006), The Strategic and Tactical Value of Commodity Futures, *Financial Analysts Journal* 62: 69-97.
- Fama, Eugene and Kenneth French (1987), Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage, *Journal of Business* 60, 55-73.
- Fama, Eugene and Kenneth French (1989), Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics* 25, 23-49.
- Fama, Eugene and G. William Schwert (1977), Asset Returns and Inflation, *Journal of Financial Economics*, 5, 115-146.
- Gorton, Gary, Fumio Hayashi, and Geert Rouwenhorst (2007), The fundamentals of commodity futures returns, Working paper, Yale University.
- Gorton, Gary and Geert Rouwenhorst (2006), Facts and fantasies about commodity futures, *Financial Analysts Journal* 62 (2), 47-68.
- Kat, Harry and Roel Oomen (2007), What Every Investor Should Know About Commodities Part II: Multivariate Return Analysis, *Journal of Investment Management* 5(3), 2007.
- Roll, Richard and Stephan Ross (1980), An empirical investigation of the arbitrage pricing theory, *Journal of Finance* 35, 1073-1103.

## Appendix: Factor Analysis

Factor model can be written as,

$$X = A + \Lambda F + U \quad (1)$$

where elements of  $U$  are normally and independently distributed with mean zero and variance covariance matrix  $\Psi$ . Under the assumption that the

factors are *i. i. d. N(0,1)* the factor representation can also be expressed in terms of the variance covariance matrix of  $X$

$$\Sigma = \Lambda\Lambda' + \Psi \quad (2)$$

where  $\Sigma$  is the variance covariance matrix of  $X$ . This representation also provides for a test of correct specification of the factor model. The null hypothesis for the adequacy of the  $k$ -factor model is.

$$H_0: \Sigma = \Lambda\Lambda' + \Psi \quad (3)$$

Covariance representation (2) also suggests that factor loadings are not unique. Let  $P$  be an orthogonal matrix ( $PP'=I$ ), then the elements of  $\Lambda P$  are different from elements of  $\Lambda$ , however

$$\Lambda P(\Lambda P)' + \Psi = \Lambda P P' \Lambda' + \Psi = \Lambda\Lambda' + \Psi = \Sigma \quad (4)$$

Different orthogonal transformations of the factor loadings can be performed to represent the same variance covariance matrix of the underlying covariates  $X$ . Orthogonal transformations of factor loadings allow the analyst to identify distinct subsets of variables in  $X$  that are closely correlated with each other. One such criterion is known as varimax rotation. This criterion strives to maximize the variance of the square of the loadings in each column of the loadings matrix  $\Lambda$ . Maximizing the variance therefore has the effect of minimizing the number of high loadings in a column, which for the purpose of our exploratory analysis will help identify commodities into groups.